Black Holes and the Quest for a Theory of Everything:

A discussion of the physical and mathematical mechanics of black holes and how they demonstrate a fundamental conflict in the search for a Theory of Everything

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Abstract:

Black holes are massive bodies of matter that exist in our universe. They have mystified and intrigued both scientists and common folk alike for many years, due to their unique properties that allow nothing to escape their enormously powerful gravitational fields (with the exception of thermal radiation). Scientists first began to ponder the notion of black holes in the late 1700s, although at that point the term "black hole" did not yet exist. At first, scientists simply surmised that if a body was massive and compact enough it could potentially have a strong enough gravitational force to entrap everything that came within its near reach – even light rays. To understand the basic mechanics of a black hole one must examine the escape velocity of such massive bodies; such analysis reveals that when the escape velocity of a massive and compact body exceeds the velocity of light the result is a body of matter that collapses into itself and gives birth to a black hole. Proper analysis of black holes must include at least an elementary level discussion of Einstein's theories of Special Relativity and General Relativity in order to understand how forces generated by such massive bodies fit into the framework of a Theory of Everything. When one examines black holes in the context of Einstein's theories it is apparent that there is no discernible difference between normal acceleration forces and gravitational forces; these forces are fundamentally related because of the nature in which massive bodies of matter warp the spacetime structure of their surroundings. The primary conclusion to be drawn from this discussion is two-fold: first, that the theory of General Relativity and the theory Quantum Mechanics mathematically contradict one another under particular, very specific physical domains – an example of which can be demonstrated by black holes. The second critical conclusion stems from the first and is that the next critical step towards developing a coherent and comprehensive Theory of Everything is the explanation and expansion of a theory of Quantum Gravity.

The conceptual foundation for the existence of black holes was first theorized in 1783 by John Mitchell (Hooft). Although Mitchell did not state that "black holes" existed, per se, he brought forth the following query to the scientific community: "are there bodies with a mass m and radius R such that $(2Gm/Rc^2) \geq 1$?" (Hooft). To put the mathematics into narrative terms, Mitchell wondered if a body could have a large enough mass and a small enough radius that its escape velocity would exceed that of the speed of light. Furthermore, he surmised that if such a body could be massive enough and compact enough to have such an escape velocity, then nothing – not even light – would be able to escape the gravitational pull of such an object (Van der Marel, et. Al.). In 1796 this notion was further investigated by Pierre Simon de Laplace.

Laplace came to ask the same question: whether an object could achieve a large enough mass that it caused “rays of light fall back towards the surface of such an object?” (Hooft). It wasn’t until 1967 that the actual term ‘black hole’ was introduced by John Archibald Wheeler (Van der Marel, et. Al.). By this time, the underlying physics that Mitchell and Laplace had first pondered had developed considerably and the mathematical understanding of black holes was much more established.

The mathematical evidence that underlies the existence of black holes is based on the fact that a body whose escape velocity is greater than the speed of light has such an enormous gravitational force pulling matter toward it that nothing can escape the boundary of the mass’s body, once within a certain distance of the object’s center of mass. To better understand this concept one should first examine the definition of ‘escape velocity.’ According to AlderPlantiarium.org the escape velocity is “the speed necessary to escape the gravitational influences of a massive body; this depends on the distance you are from the center of the body and the mass of the body” (Teacher resources). In mathematical terms a massive body’s escape velocity v from a distance r from the center of gravity of an object with mass m, is defined as: $\frac{1}{2}(v^2) = (Gm)/r$, this according to Sir Isaac Newton’s theory of gravity (Hooft). Rewritten to isolate “v”, we see that the escape velocity, v, is: $v = \sqrt{(2Gm)/r}$.

The realization that a massive and very compact body can mathematically have an escape velocity that is greater than that of light, leads to several very consequential results: the first being that a body this massive will have an incredibly strong gravitational pull and will forcefully attract matter that is near it; the second, and more important, consequence is that there will exist a border around the mass at which point any matter that is attracted past this border will no longer be able escape the mass and will become, itself, a part of the mass. This border occurs at the distance *r* from the center of the mass at which the gravitational pull is so strong that the escape velocity is greater than the speed of light; this distance is called the Schwarzchild radius, named after the German physicist who quantified the distance in mathematical terms (Hooft). This theoretical border is called an "event horizon" and has a very important implication, as far as the existence of black holes is concerned. The event horizon is essentially the point of no return. In other words, it is the boundary surrounding a black hole at which point matter will be forever stuck within the black hole, lumped together with the rest of the body's mass – not actually forever, because energy does eventually dissipate back out of a black hole through an incredibly slow process of very weak thermal radiation (Hooft). According to Roeland Van der Marel, the principal investigator (author) of the HubbleSite website, "the event horizon is an imaginary sphere that measures how close to the singularity you [a particle of matter] can safely get. Once you [the particle] have passed the event horizon, it becomes impossible to escape: you will be drawn in by the black hole's gravitational pull and squashed into the singularity" (Van der Marel, et. Al.).

A discussion of a massive body's event horizon brings forth another important issue related to black holes: namely, how does matter within a black hole behave? The reality is that scientists do not yet know exactly what occurs within a black hole, because technology has not developed to the point at which observation of black holes at such a close distances is possible; furthermore, even if such a tool or technology was created, we would still have trouble tracking it and learning from it, because once the piece of equipment entered a black hole it would be unable to emit electromagnetic waves to communicate information back to Earth (Van der Marel, et. Al.). Nevertheless, scientists have hypothesized about the behavior of matter within a black hole, based on mathematical inferences. It is thought that when a star (or other body of mass large enough and compact enough to form into a black hole) dies and collapses into a black hole after supernova, its mass is compacted into an infinitely small bunch of matter that is gathered at a single point in the very center of the black hole. This point of infinitely small and compact matter is referred to as the black hole's 'singularity' (Van der Marel, et. Al.). Scientists also hypothesize that once matter crosses the event horizon of a black hole it loses all of its distinguishing features and is essentially lumped together with the rest of the matter that makes up the mass of the black hole. According to HubbleSite "black holes themselves are all identical, except for three characteristic properties: the mass of the black hole (how much stuff it is made of), its spin (whether and how fast it rotates around an axis), and its electric charge. Amazingly, black holes completely erase all of the other complex properties of the objects that they swallow" (Van der Marel, et. Al.). The manner in which black holes consume surrounding matter and nullify any of its variance is a fascinating reality about these super massive and compact bodies. Although "current theories predict that all the matter in a black hole is piled up in a single point at the center, we do not understand how this central singularity works" – this according to HubbleSite's author, Mr. Marel. "To properly understand the black hole center requires a fusion of the theory of gravity with the theory that describes the behavior of matter on the smallest scales, called Quantum Mechanics. This unifying theory has already been given a name, Quantum Gravity, but how it works is still unknown" (Van der Marel, et. Al.).

Black holes can be understood from a narrative perspective by simply viewing the phenomenon as the result of a hugely massive and compact body continuously collapsing on itself, due to its enormous gravitational force; however, this does not put the issue into proper perspective, in terms of its role in the larger scheme of the fundamental forces of the universe and the search for a Theory of Everything. In order to discuss black holes on this broader fundamental level, one must bring Einstein's theories of Special and General relativity into the discussion. Einstein's first of the two theories, the Theory of Special Relativity "describes how matter moves through time and space and its predictions have been verified to more than twenty decimal places of accuracy" (Anissimov: special relativity). Special Relativity is based on two basic postulates: the first being that the laws of physics are the same regardless of absolute velocity and the second being that the speed of light is constant for all observers (Anissimov: special relativity). The basic notion of Special Relativity is that there does not exist a constant structure of time and space; rather time and space behave as one four-dimensional structure referred to as "spacetime" and that both space and time are different to each observer (Anissimov: special relativity). Special Relativity is needed to understand the bigger picture – namely, the hierarchy of theorems that attempt to describe and unify the fundamental forces of nature. Special Relativity is of particular importance in understanding this, because it allows one to grasp a basic understanding of General Relativity, which in turn sheds light on how the mechanics of black holes fits nicely into certain parts of the unifying hierarchy, yet fundamentally conflicts with other parts.

Einstein's theory of General Relativity is an expansion of his theory of Special Relativity. It examines similar concepts of relativity and a four-dimensional spacetime structure, as they apply to very large objects or bodies. The basic postulate of General Relativity is that "matter deforms the geometry of spacetime, and spacetime deformations cause matter to move, which we see as gravity" (Anissimov: general relativity). Essentially, this amounts to the unification, or at least unified causation, of two fundamental forces in nature: namely acceleration and gravity. In his theory of General Relativity Einstein explains that force caused by gravity and forced caused by other forms of acceleration are indistinguishable to the observer on whom the force is acting.

“This principle, that all physical laws are the same for accelerated observers and observers in a gravitational field, is known as the equivalence principle; it has been experimentally tested to more than twelve decimal places of accuracy” (Anissimov: general relativity). The importance of General Relativity as it applies to one’s understanding of black holes is that the enormous attractive force between a black hole’s central mass and surrounding matter is perceived as a gravitational force, but is, in reality, simply an acceleration force originating from spacetime deformations that cause matter to move. As is apparent from this description, both matter and spacetime act upon each other and cause changes in the perception of one another – and these changes in perception are relative to the spacetime structure in which the observer perceives them.

Einstein’s theories of Special Relativity and General Relativity are critical to this analysis of the mechanics of black holes, because they shed light on a more universal issue: namely, that the existence of black holes demonstrates a specific physical domain in which a theory of Quantum Gravity is needed to keep the laws of physics, as they are currently known, from contradicting each other. However, to understand the difficulties posed by the quest for a theory of Quantum Gravity (and ultimately a Theory of Everything), a discussion of limits is required. Mathematical limits are the basis of Calculus, which is used to describe many physical events, such as the instantaneous rate of change and the area under a curve. To give a purely mathematical example, consider what happens to the value of a function of x equal to ‘one divided by x’, as the value of x approaches infinity: the result is one divided by an infinitely large number – the limit of which is zero. Now consider the limit of the same function of x, however as x approaches zero. The resulting limit is undefined, because one divided by an extremely small (but positive) number (i.e. 1 x 10 raised to the negative 100^{th} power) is equal to an extremely large number; thus, as x approaches zero, the limit approaches infinity, which is by definition boundless, and therefore undefined. To bring the discussion back to the topic of black holes, consider the following excerpt: “The gravitational force is strong near a black hole because all the black hole's matter is concentrated at a single point in its center…Physicists call this point a singularity” (McClintock). To most people this statement by Jeffrey E. McClintock, (Ph.D.) the Smithsonian Astrophysical Observatory’s Senior Astrophysicist, may seem benign and void of the concept of limits. However, to the mathematician, it is apparent that “a single point” is volume-less, because it is dimensionless and occupies no space.

Further demonstration of the inherent nature of limits to a theory of Quantum Gravity can be seen in this statement on NASA’s Worldbook website: “The surface of a black hole is known as the event horizon. This is not a normal surface that you could see or touch. At the event horizon, the pull of gravity becomes infinitely strong. Thus, an object can exist there for only an instant as it plunges inward at the speed of light” (McClintock). Once again this statement is saturated with the implications of mathematical limits. If the pull of gravity becomes “infinitely strong” then the attractive force of the singularity upon matter within the event horizon increases without bound, as the matter gets closer and closer to the center of mass. In order for the gravitational force to increase, the square of the distance between the center of gravity and the object must decrease. Thus, if the gravitational force increases without bound, the limit of the distance between the central singularity and matter attracted from outside the black hole inward and past the event horizon must mathematically be zero (conceptually similar to the limit of one divided by x, as x approaches zero). Now, if one follows the mathematical logic of limits, as they apply to black holes, and then juxtaposes this conclusion with our current knowledge of atomic and quantum structure, the result reveals the clear and obvious difficulty with a theory of Quantum Gravity, if based solely on our current knowledge and understanding.

To more clearly demonstrate this contradiction, let us review the details of three important mathematical equations and then apply limit theory to see how these equations behave under certain physical conditions. Consider the three equations and limit applications listed below:

1. Gravity formula: $F_{gravity} = GM_1M_2/r^2$

Consider: $\lim r \to 0^+$, $GM_1M_2/r^2 = +\infty$ **(Equation 1)**

2. Density formula: Density = mass/vol

Consider: $\lim vol \to 0^+$, $mass/vol = +\infty$ **(Equation 2)**

3. Escape Velocity: $v = \sqrt{(2Gm/r)}$

Consider: $\lim r \to 0^+$, $\sqrt{(2Gm/r)} = +\infty$ **(Equation 3)**

Notice that as the gravitational force ("$F_{gravity}$" in Equation 1 above) of the black hole attracts matter closer to the center, the radius ("r" in Equations 1 and 3) between the singularity and the particular matter at hand decreases. As "r" gets smaller, we see from Equation 1 above, that the $F_{gravity}$ acting on a given particle of matter becomes stronger, thus sucking it even closer to the singularity, and in turn, further reducing "r". The resulting limits of Equations 1 and 3 above, when taken as r approaches zero (from the positive side), dictates that this process of an increasing gravitational force caused by – and in turn causing – a reduced "r", becomes a vicious cycle with any and all matter within the event horizon, until eventually each particle of matter that crosses the event horizon is collapsed into the infinitely small singularity.

Another result of this vicious cycle that is clearly visible from Equation 2 above, is that as $F_{gravity}$ increases, causing "r" to decrease, the volume of the inner sphere containing the black hole's mass decreases constantly and continuously (a geometrically defined result of "r" decreasing), thus causing the density of this inner mass to increase without bound as well. In reality this process described above (of a shrinking inner mass) happens initially – and almost instantaneously – when the black hole is first formed. From that point forward, mathematics suggest that the singularity remains infinitely small in volume (volume-less) and infinitely dense. After the black hole is "born" in this supernova process, surrounding matter near the black hole continues to be drawn in and the process of increasing gravitational attraction cycling with a reduction of "r" continues indefinitely with each particle of matter that crosses over the event horizon threshold – mathematically resulting in the entire mass within the event horizon piled up into a singularity, with an infinitely small volume.

That said, black holes are thought to have defined masses, as scientists have been able to approximate the masses of particular black holes in our galaxy based on the level to which they cause the light passing nearby to bend (McClintock). In light of this, consider the following: density is formally defined as "mass per unit volume," or an object's mass, divided by its volume (Ophardt). Coupling this definition of density with what was previously concluded about black holes, and one ends up at the following logical conclusion: the singularity has an infinitely small volume, yet a defined (albeit constantly growing) mass, and thus an infinitely large density.

To bring the discussion back to an explanation of why a theory of Quantum Gravity has so many challenges, consider the following definition of matter: "Matter is any substance which has mass and occupies space. All physical objects are composed of matter, in the form of atoms, which are in turn composed of protons, neutrons, and electrons" (Jones). Furthermore, McClintock states the following on NASA's website, regarding the dimension of a black hole's singularity: "Physicists call this point a singularity. It is believed to be much smaller than an atom's nucleus" (McClintock). Now the inherent contradiction between Einstein's Theory of General Relativity and existing theory of Quantum Mechanics becomes apparent: General Relativity mathematically dictates that all of the matter contained within a black hole is concentrated into a single, volume-less point with a defined mass and an undefined, infinite density, yet Quantum Mechanics tells us that the distinguishing feature of matter is that it has a defined mass and a defined volume.

At this point one has reached the crossroads of modern scientific understanding, where General Relativity and Quantum Mechanics cease to coexist in a mathematically symbiotic manner. One can see the issue in simple terms, because an atom cannot exist in a stationary state, without taking up space – and therefore having volume; yet General Relativity wants the atom to be able to exist without volume in certain mathematical, yet still theoretically physical domains. The solution to this paradox lies in the theory of Quantum Gravity, which currently is but an empty shell – the framework of an unknown body of knowledge that is believed to exist, but not known how it exists. Quantum Gravity is the merging of two bodies of contradicting scientific knowledge awaiting a mathematical solution that will allow both knowledge bases to coexist within all relevant and theoretically physical domains.

To conclude, the implication of Einstein's theories of Special Relativity and General Relativity as they pertain to black holes is that black holes are huge conglomerations of matter that are massive enough to cause the spacetime structure surrounding them to warp considerably; this warping of spacetime around a black hole is, in turn, significant enough to cause movement in the matter surrounding a black hole – and it is the force behind this particular movement of surrounding matter that is perceived as the enormous gravitational force of the black hole. Once this premise is understood, the functional mechanics of a black hole can be viewed in the context of the bigger picture as they relate to and coexist with several of the fundamental forces of the universe. This discussion of black holes is ultimately just a demonstration of one specific theoretical physical domain in which General Relativity and Quantum Mechanics are mathematically incompatible. By using this analysis of black holes to identify this particular domain, the following conclusion is generated: one of the most important steps left to further our understanding of a Theory of Everything is to understand the mathematical mechanics that unify Quantum Mechanics and General Relativity (in light of gravity being derived from the theory of General Relativity) into a theory of Quantum Gravity. Once this theory of Quantum Gravity is defined and explained, humanity will have solved one of the key scientific mysteries that has long eluded the realm of human knowledge, and we will be one step closer to an ultimate Theory of Everything.

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